

extensive survey, "Continued fraction representations for functions related to the Gamma function," by L. J. Lange, gives a number of continued fraction representations for a large variety of specialized complex functions which are related to the classical gamma function. Between these two expositions of generalization and specialization, the remaining papers included here present new results covering convergence questions and truncation error bounds for continued fractions and sequences of linear fractional transformations, applications to moment problems, orthogonal functions on the real line and the unit circle, Szegő and related polynomials, their zeros and applications to frequency analysis. In addition, the preface contains an interesting historical survey of the Colorado-Norway group which began with Wolf Thron and his early association with the late Arne Magnus.

T. S. CHIHARA

Department of Mathematics, Computer Science & Statistics
Purdue University Calumet
Hammond, IN 46323-2094

28[68-01, 68Q40].—RICHARD ZIPPEL, *Effective Polynomial Computation*, Kluwer, Boston, 1993, xii + 363 pp., 24 cm. Price \$87.50.

The field of computer algebra has gained widespread attention in recent years with the increased popularity and use of computer algebra systems such as Derive, Maple and Mathematica (and others) in the general scientific community. These systems have an extensive set of mathematical capabilities in such diverse areas as basic algebra (e.g., polynomial operations such as factorization and GCD computation) and analysis (e.g., determining closed-form solutions of integrals and differential equations). For mathematicians using these systems there is often a natural desire to learn more about the algorithms that are used in these systems. Unfortunately, there are very few suitable textbooks or survey articles that introduce mathematicians to these algorithms. Since the alternative is to search through a wide variety of research papers and Ph.D. theses, this makes the area a difficult one to begin research.

This text is meant as a one-semester course to introduce students and researchers to some of the fundamental algorithms used in computer algebra, in particular, for computation with polynomials. It can be used as a text for upper-year undergraduate or starting graduate students. The term "effective" used in the title could also be read as practical in the sense that the approach used is one of describing algorithms that work in practice, rather than only in theory.

The author makes the point that many of the algorithms of polynomial computation have their origins in computational number theory (e.g., Hensel lifting) and uses this as his starting point. The topics covered include continued fractions, solving Diophantine equations, and algorithms for polynomial computations such as factorization, interpolation, elimination and computation of greatest common divisors. Computational methods such as Chinese remaindering and Hensel lifting that are used to overcome the basic problem of intermediate expression swell are also covered. In addition to these deterministic methods the author discusses some heuristic and probabilistic techniques used in some computer algebra computations.

A point should be made that many of these algorithms are not what a typical mathematician would expect or has been taught. Indeed, this is one of the facts that makes this such an interesting field of study. As an example, consider the simple problem of computing the greatest common divisor of say two multivariate polynomials (a very common operation required by many algorithms in computer algebra). Every undergraduate mathematics student learns that the way to compute this would be to use Euclid's algorithm. It will come as a surprise to many that this is one of the last methods that one would want to use in this case. For example, this method has the problem that there is significant growth in the size of coefficients during the intermediate steps. Instead, one can use this text to learn the methods which are used in practice. The text includes in various chapters: polynomial remainder sequences; heuristic gcd computation; modular methods based on the use of Chinese remaindering; Hensel methods using p -adic arithmetic; and probabilistic methods. The last-named method includes the well-known "sparsemod" algorithm that came from the author's own Ph.D. thesis from the days when he was associated with the Macsyma project at MIT.

This book will be a valuable resource for anyone interested in pursuing research in the field of computer algebra. It is well written and easy to read, with good use made of examples to illustrate the mathematical difficulties that are encountered in polynomial computation. The book has numerous historical references throughout the text in addition to the interesting historical notes that are found at the end of each chapter. It does not cover the same variety of computer algebra topics as found in, for example, [1] but it does go into much more depth in the topics that it does discuss. My only complaint with the text is a minor one—I would like to have seen exercises at the end of each chapter. This would make it easier to use in an upper-level undergraduate course on computer algebra.

GEORGE LABAHN

Department of Computer Science
University of Waterloo
Waterloo, Ontario N2L 3G1
Canada

1. K. O. Geddes, S. R. Czapor, and G. Labahn, *Algorithms for computer algebra*, Kluwer, Dordrecht, 1992.